Supersonic Nozzle Design and Analysis

For a Compact, high-Mach Number Wind Tunnel

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A preliminary design and analysis of a converging-diverging nozzle was performed. The diverging contour was generated using the method of characteristics. Design evaluation was carried out using high-fidelity simulations. The nozzle is to be used in a compact, high-Mach number wind tunnel for the purpose of material characterization.

Motivation & Requirements

- Preform highly diagnosed material characterization experiments on coupon sized samples
- Compact size will allow for rapid turnaround of results by reducing experimental setup time with advanced diagnostics
- $6 \leq M_{test} \leq 12$
- Nozzle length < 1 meter

Background & Approach

- How is flow accelerated to supersonic speeds?

$$\frac{dA}{A} = -\frac{dV}{V}\left(1 - M^2\right)$$

$$M = \frac{V}{\sqrt{\gamma R T}}$$

- Must use a converging-diverging nozzle
- Need a nozzle that produces smooth, shock-free flow at a specified Mach number
- Implement a method of nozzle design to produce desired flow
- Computationally verify nozzle performance

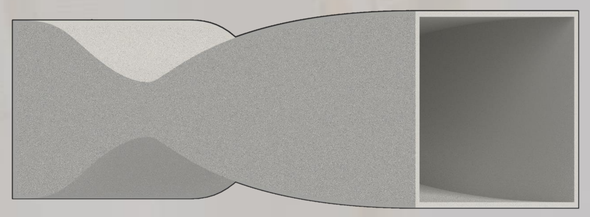

Figure 1. Supersonic, planar nozzle

Method of Characteristics

- Flow is accelerated via expansion waves centered at a sharp throat
- Weak waves (turning angle $\Delta\theta$) incident on wall generates a reflected wave at same angle to preserve wall boundary condition [1]
- Turning the wall by angle $\Delta\theta$ eliminates reflected wave

$$\frac{\partial^2\phi}{\partial^2 x^2} + \frac{\partial^2\phi}{\partial^2 y^2} - \frac{1}{a^2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2\frac{\partial^2\phi}{\partial^2 x^2} + \left(\frac{\partial\phi}{\partial y}\right)^2\frac{\partial^2\phi}{\partial^2 y^2}\right] - \frac{2}{a^2}\left[\frac{\partial\phi}{\partial x}\frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y}\right] = 0$$

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}}\,tan^{-1}\sqrt{\frac{\gamma+1}{\gamma-1}(M^2-1)} - tan^{-1}\sqrt{M^2-1}$$

$$\theta + \nu(M) = constant = C^+$$

$$\theta - \nu(M) = constant = C^-$$

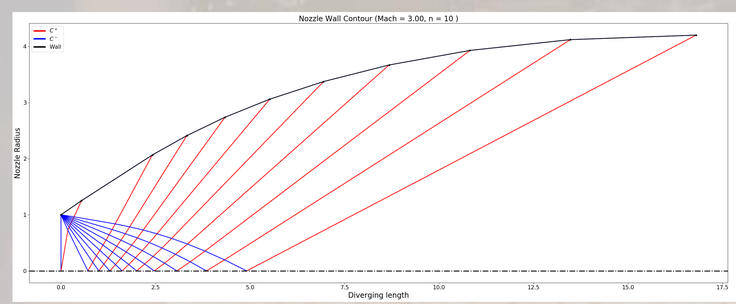

Figure 2. Mach 3 nozzle contour via method of characteristics (n =10)

Nozzle Analysis

- 2-D, steady, inviscid flow, adiabatic walls
- Inlet: 36.74 [atm], 840 [K]
- Outlet: 1 [atm], 300 [K]

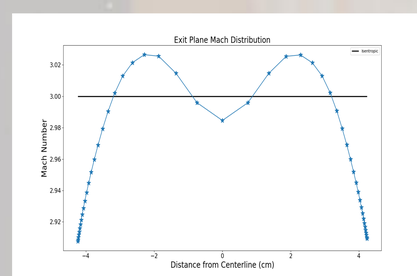

Figure 3. Mach number distribution in exit plane

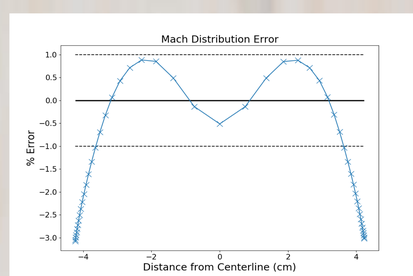

Figure 4. Mach number error in exit plane

Results

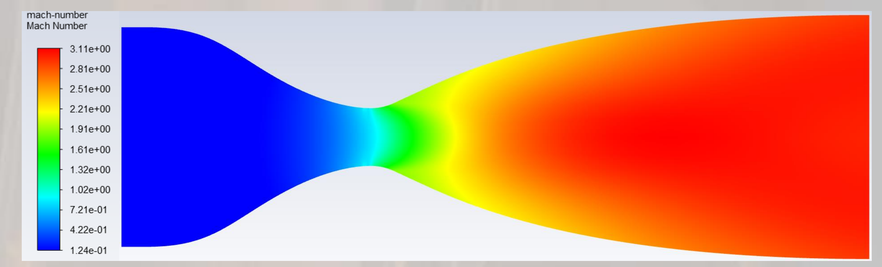

Figure 5. Nozzle inviscid flow simulations displaying Mach contours

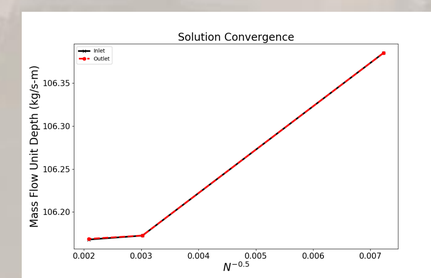

Figure 6. Asymptotic convergence of mass flow rate

- Solution convergence using three mesh densities: 19k, 110k, 230k elements
- Mach number smoothly increases throughout nozzle
- Exit plane Mach distribution has slight deviations from isentropic value
- Maximum error of 3% near walls

Conclusions

- Implemented a method that allows for rapid generation of nozzle designs
- Verified that nozzles are produce a realistic flow field
- Centerline Mach number yet to match expected value

Future Work

- Modify method of characteristics code to produce gradual expansion nozzle & account for viscous effects
- Evaluate nozzle flow field at higher Mach numbers with Stanford's high fidelity SU2 code
- Analyze walls loads during operating conditions using ALE3D to determine structural integrity requirements

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References

[1] Anderson, J. D. (2003). *Modern compressible flow: With historical perspective*. Boston: McGraw-Hill.

Background Image - Ryan Chylinski (SpaceFlightInsider.com)

Simulation analysis validates method of characteristics for nozzle design

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